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| **Lesson Title** | **Using Order of Operations to Evaluate and/or Simplify Expressions** |
| **Subject area/grade level** | **Mathematics Grades 5/6** |
| **Introduction** | A major focus of pre-algebra courses is the familiarize students with variable expressions. This lesson is a bridge between simplifying numerical expressions and evaluating algebraic expressions. |
| **Lesson Length** | 2 class periods (depending upon prerequisite knowledge and prior understandings) |
| **Materials** | Index cards (Alternative: Loaf of bread, jar of jelly, knife, plate)  For *Target Number* set of 5 number cubes per small group & game sheet  For *Order Please* one set of 5 number cubes per class; Order Please Chart for projector or board  TI-73 calculator (optional) |
| **Lesson Overview** | The lesson applies/extends the order of operations with practice in a game context. Together with property of substitution, the order of operations is used to evaluate a variable expression.  Online applets (NCTM Illuminations) and technology (TI-73) are employed to deepen understanding about substitution and evaluation of expressions.  Training Discussion: Role of Technology in the Elementary/Middle Math Classroom  OPTION 1:Respond to the question in the excerpt below.Top of Form Then discuss Cathy Seeley’s response.  **From Algebraic Thinking chat with former NCTM President Cathy Seeley** NCTM.org  **Question:** Do you feel that calculators should be used on a daily basis in middle school math classes?  **Cathy Seeley:** I think calculators should be available for students to use at these grades, understanding that the teacher helps students decide when and how to use them. It is critical that students learn to make these decisions. Teachers can help by identifying when calculators should not be used ("Put your calculators away; we're going to do some mental math.") and when they can be helpful (as in solving complex problems). We also need to make sure that we capitalize on the availability of calculators by giving students challenging problems that go beyond the limitations of what we can do with students when they do not have such access. Simply giving students long lists of computational exercises and then giving them calculators to do them defeats the purpose of having this tool available.  OPTION 2: Divide participants, give each group/individual a document. Participants use interactive markings (~ = I’m not sure about this, + or - = I agree or I disagree; ! = aha moment for me; ? = I need more information) while reading a document concerning calculator use in elementary/middle grades.  **NCTM Principles and Standards for School Mathematics: Technology Principle** <http://www.nctm.org/standards/content.aspx?id=3448>  **Middle School Math and the Calculator (When to Use a Calculator)**  Dr. Louise Johnson <http://www.iched.org/cms/scripts/page.php?site_id=iched&item_id=math_calculator>  **Agenda for Action Recommendation 3:Technology**  <http://www.nctm.org/standards/content.aspx?id=17282>  Educator’s opinion |
| **Tennessee Standards**  **MCS Power Standards** | **TN GRADE 5**  GLE 0506.3.1 Understand and use order of operations.  GLE 0506.3.2 Develop and apply the concept of variable.  GLE 0506.3.3 Understand and apply the substitution property.  SPI 0506.3.1 Evaluate algebraic expressions involving decimals and fractions using order of operations.  SPI 0506.3.2 Evaluate multi-step numerical expressions involving fractions using order of operations.  **MCS Grade 5**  Solve multi-step problems using whole numbers, mixed numbers, decimals and fractions.  **TN Grade 6**  SPI 0606.3.2 Use order of operations and parentheses to simplify expressions and solve problems.  SPI 0606.3.4 Rewrite expressions to represent quantities in different ways.  0606.1.11 Model algebraic expressions with manipulatives, technology, and pencil and paper.  SPI 0606.1.5 Model algebraic expressions using algebra tiles.  GLE 0606.3.2 Interpret and represent algebraic relationships with variables in expressions, simple equations and inequalities.  0606.3.3 Recognize the use of juxtaposition (such as 3x, ab) to stand for multiplication, and the convention in these cases of writing numbers before letters.  0606.3.7 Move fluently between different representations (such as verbal, tabular, numerical, algebraic, and graphical) of equations and expressions.  SPI 0606.3.5 Translate between verbal expressions and algebraic expressions.  **MCS Grade 6**  Use expressions, equations and formulas to solve problems.  Write and solve two-step equations and inequalities.  Interpret and represent algebraic relationships with variables in expressions, simple equations and inequalities. |
| **Lesson objective(s)** | The Learner will   * correctly verbalize order of operations * apply order of operations when simplifying/evaluating expressions * use correct notation to write a numerical expression * substitute a value for a variable in an expression using grouping symbols appropriately to retain order of operations * use technology with appropriate notation to evaluate expressions   Training Discussion: “I Can…” Statements (expressions/equations) |
| **ENGAGEMENT**   * Describe how the teacher will capture students’ interest. * What kind of questions should the students ask themselves after the engagement? | |  |  |  | | --- | --- | --- | | Ask students to arrange index cards, each of which contains one step of an everyday process for which order of steps is unique. | | | | Grocery Shopping | Build a house | Wrap a gift | | Drive to store  Park car in store parking lot  Exit car  Walk to store entrance  Get a cart to roll thru store  Fill cart with groceries  Roll cart to check out  Place cart items on counter  Pay for groceries  Put bags back into cart  Exit store  Walk cart to car  Put groceries in car  Get in car to go home | Get materials  Make foundation  Build floor  Make frame for walls  Add roof  Add windows and doors  Add electrical lines/water pipes  Insulate walls  Cover wall studs with drywall  Paint walls | Secure item for gift  Remove price tag  Place item in box  Cut paper to fit box size  Wrap paper around box  Fold paper to box shape  Tape folds of paper in place  Wrap ribbon around box  Tie ribbon in bow  Add card to top of box |   OR  Ask students to write down instructions to make a jelly sandwich; take them up and select one to follow. Follow instructions extremely literally and expect to make a mess. But take suggestions on how the directions should be amended to explicitly describe actions. Discuss “implied instructions” (such as, get bread implies you’ll have take the closure off the package, reach in and take a piece out) and “conventions” (such as putting jelly on bread requires spreading with a knife).  When does order of actions matter in math?  What kind of math problem has multiple steps?  How and when do we write action (operation) steps for computations?  Explain that we have need for these same types of communication and understanding in mathematics, i.e. there is an order to operations so that everyone gets the same value when evaluating a numerical statement. |
| **EXPLORATION**   * Describe what hands-on/minds-on activities students will be doing.      * List “big idea” conceptual questions the teacher will use to encourage and/or focus students’ exploration | Present one or more numerical/variable expressions including multiple operations and ask students to determine a number that they believe is the same as the given expression and to write that value on an index to be handed to the teacher  **Simplify 14 – 6 x 4 – 2 (20 ÷ 5 – 1) Evaluate 5ab2 – 4a/2 + b when a = 12 and b = 4**  Before you begin as a class to look at the values, **(HOT)** ask students to make observations (record their observations on the board) about each expression one at a time and to make comparisons between them. **(CQ)** Can they identify which is called a numerical expression and which is called a variable (algebraic) expression?  List class members’ values on the board. In case all the answers are correct, have a few cards ready with answers gained by incorrect order of operations so you can indicate the importance of all using the same “conventions” or order of operations. Ask volunteers to show how the value might be different depending upon which operation is performed first, second, third, etc.  **(CQ)** Conclude with the question: Which one is correct?  Students have some experience with order of operations. Ask students what order of operation rules they already know. Generate a list on the board.  Follow up by adding precision to directions similar to how the directions to ‘make a jelly sandwich’ steps in process were clarified. For instance, if a student says MDAS, clarify M is not before D but that are done in the order in which they occur when moving form left to right in the expression.)  As a whole class or in small groups begin to develop a **Graphic Organizer for Order of Operations** that they have already experienced (flow chart, linear array, outline).  **(CQ)** Ask: What about those expressions we have discussed is confusing to you? What do you see in this expression that isn’t covered by the rules we already know? Generate a list of issues not addressed by rules they already know (e.g. unknown operation symbols; grouping symbols; exponents; change in symbols to accommodate a variable changed to a numeral (substitution), etc.)  Big Idea: We need an order of operations that is consistent so we will know what value an expression has.  **(CQ)** Conceptual questions embedded throughout scripted lesson. |
| **EXPLANATION**   * Student explanations should precede introduction of terms or explanations by the teacher. * What questions or techniques will the teacher use to help students connect their exploration to the concept under examination? * List higher order thinking **(HOT)** questions which teachers will use to solicit *student* explanations and help them to justify their explanations. | **(CQ)** Ask students to explain their understanding of terms: **evaluate, simplify**.  **(HOT)** Generalize common notions.  Revisit problems:  Simplify 14 – 6x4-2(20÷5-1)  Evaluate 5ab2 – 4a/2 + b when a = 12 and b = 4  Deal with identified issues: unknown operation symbols; grouping symbols; exponents; change in symbols to accommodate a variable changed to a numeral (substitution), etc.  **(HOT)** Are there some “implied instructions” as discovered when making the jelly sandwich to operations implied by mathematical notation (i.e. m/5 means m x 1/5 or m÷5; implied multiplication: 3n means 3 times n and the number in a position like 3 is called a **numerical coefficient**; (4)(5) means 4 x 5) and “conventions” (i.e. writing coefficients before variables in a term, writing variable factors in alphabetical order.)  **(HOT)**: Summarize: What can we change or add to our (student generated) ‘order of operations list’ to establish a comprehensive set of rules?  **(HOT)** Ask students for suggestions to refine order of operations **graphic**.  Apply caution when students say PEMDAS: identify and clarify issues inherent in the PEMDAS acronym for order of operations. Immediately use established order of operations to simplify the given expressions to one value.  **(HOT)** Develop understanding of the term **expression**. (Concept card, Math journal vocabulary page) Add new terms as identified in lesson to Word Wall and incorporate them into word wall activities. (Training Handouts)  **(OE)** Student partners together on 2-3 practice problems (from text or board) while teacher circulates. |
| **ELABORATION**   * Describe how students will develop a more sophisticated understanding of the concept. * What vocabulary will be introduced and how will it connect to students’ observations? * How is this knowledge applied in our daily lives? | Training Discussion: Agree or not? “Games/competition peak student interest.” Identify PROS/CONS.  **(HOTS)** These activities give students practice writing numerical expressions using math symbols. Students develop strategies.  **(OE) Order Please?** From TN DOE Gateway Training Manual, Phase I (Game board handout)  In this activity, teams collaborate as they compete to be the first to cover four numbers in a row on a game board. Students will create a numerical expression that equals one of the numbers on the game board using a set of numbers gained from rolling three number cubes. Teams must present their CORRECTLY written and simplified expression before they can use their teams’ marker to cover a number.  **(OE) Target Number: construct numerical expression to compute given value or one close to it.**  In this activity, students will have a target number and a set of numbers that they can use to try to hit (or get close to) the target number. Score on each round is the difference between target number and value of constructed expression. Lowest total score wins. (Game sheet with instructions handout.)  Notes to teacher: In both activities, choice and order of operations are student decisions. However, it is not as easy as it sounds! Quite often students will be able to verbalize what they will do with the set of numbers to get the target number but it takes some practice for them to be able to put the symbols together so that the expression, following the order of operations, actually equals the number they intend for it to equal. Be sure to work enough examples as a whole group before you let them play the game. If one person hits the target number very quickly, **(HOT)** ask if students can think of other ways to hit the same target with the same set of working numbers. As they give their ideas, let them verbalize the order of operations and describe the notation they will use. Keep working to get the expression to contain all required numbers and get close to the target number. Emphasize that the objective of this game is to get a low score!  **The following examples given in terms of the Target Numbers activity apply to both activities.**  **Address the issue of getting all numbers into an expression with this example**.  **Example:** Target number = 41 Working numbers {3,3,5,2,1}  One way to get the 41 is to number 6 and 7, multiply them, and subtract 1 appropriately written as  (3+3)(5+2)-1 = 41  Beginners may write 3+3=6 and 5+2 = 7 and 6x7=42 and 42-1 = 41 which is not **a numerical expression** containing all five working numbers. You will have to help them include all five numbers and operations into one expression.  **Address the issue of improper use of the = sign with this example**  **Example:** Target Number = 31 Working numbers {4,3,3,3,5}  A beginner may write 3+3=6x5=30+4=34-3=31. The operations used to put the number together are correct but **the use of the string of equals signs is not correct**. This statement actually says that 6=30=34=31. Remind students that we have a way to circumvent the MDAS order i.e. use of parentheses.) **(HOT)** Ask students for an improvement on plan shown in the example.  **Help students develop mathematical concepts as well as strategies as they construct expressions with this example.**  **Example:** Target number = 18 Working numbers {4,2,2,3,6}  It is possible to get the target 18 using only two of these numbers 3 x 6. One strategy is to figure out a way to make the rest of the numbers equal 1. **(HOT)** Ask students how that will help – if you multiply the expression that equals 1 by the existing value, the answer will not change, i.e. 1 x 18 = 18. (1 is called the **identity element for multiplication** because when you multiply a number by 1 you get a product **identical** to the number you multiplied it by.) Ask: How can we do this?  Another strategy is to figure out a way to make the rest of the numbers equal 0. **(HOT)** Ask students how that will help – if you add the expression that equals 0 to the existing value, the answer will not change, i.e. 0 + 18 = 18. (0 is called the **identity** **element for addition** because when you add 0 to a number, you get a sum **identical** to the number you added it to.) Ask: How can we do this?  Vocabulary (see use in examples): numerical expression, variable (algebraic) expression, numerical coefficient, evaluate, simplify; meaning of mathematical symbols, equality and equal sign, identity element for addition/multiplication |
| **EXTENSION**  **INVOLVING TECHNOLOGY** | Trainer Demonstration: Create Expressions Using Algebra Tiles  Use Substitution to Evaluate Expressions with Algebra Tiles  <http://illuminations.nctm.org/ActivityDetail.aspx?ID=216>  Evaluate expression on graphing calculators [handouts: *This is a Test* (TI-73), *Evaluating Expressions* (TI-84)]  Have students determine if a scientific calculator or four-function calculator uses order of operations. |
| **EVALUATION**   * How will students demonstrate that they have achieved the lesson objective? * This should be embedded throughout the lesson as well as at the end of the lesson | **(OE) denotes Opportunities for Evaluation embedded in exercises, games, and activities.**  Students can demonstrate correct application in traditional format in board work, homework, quiz, warm-up in succeeding class.  **Alternative Opportunities for Evaluation**  A. SNATCH: Practice with whiteboard/dry erase marker (provides immediate feedback.)  B. Centers: Students collaborate in pairs or small groups to complete one or more tasks at each center Select any of the following ideas or develop your own.   1. Task: Manual application of order of operations involving whole numbers 2. Task: Manual application of order of operations in problems involving fractions, decimals, or mixed numbers. 3. Task: Manual evaluation of variable expressions using substitution and order of operations. 4. Task: Using provided TI-73 or TI-84 calculators, check work from one of previous centers. Write about errors you discovered. 5. Task: Simplify expressions to decode answer to a riddle (see Algebra with Pizzazz) 6. Task: Hands-on evaluation of expression using set of algebra tiles. 7. Task: Generate a worksheet to be used with VersaTiles™ (ETA/Cuisenaire (Automatic check by creation of correct tile pattern) 8. Task: Computer (one or more) available for students to use applet in NCTM Illuminations (demonstrated in training) 9. Task: Correct and explain in writing error(s) you can find in the work shown. 10. Task: Use one of your names (first, middle, last) or some combination of your names so that you have at least 5 letters. Assign letter values (a=1, b=2, c=3, etc.) and use those values to create a numerical expression that will simplify to your age. (If you can’t get exactly your age, get close as close to it as you can.) 11. Task: Use the numerical expression you created in the previous center. Substitute the letters in place of the numbers in the numerical expression to get a variable expression that will simplify to your age (or to a number close to your age).   C. Individual evaluation  *Make the Number* (handout), *Out of Order* (handout), 1-2-3 *Order Me Please* (handout) |